A STUDY OF THE CHAIN-RELAXATION METHOD OF NETWORK ANALYSIS

BY

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Contents

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I. Introduction.

(A) In the analysis of electrical circuits of the type commonly encountered in radio and communication devices, a problem of major importance is the choice of method by which the solution of a network is to be obtained. There are in general use today two principal methods, each having certain advantages and disadvantages for the different types of network. A need has long existed for a single method which would be universally applicable, or nearly so, and which would have the added advantage of reducing the effort required for solution. In a recent article (Bibliography 1), the "Chain-Relaxation" method has been presented as a possible fulfillment of this need. It is the purpose of this paper to illustrate the application of this new method of analysis as a means of comparison with the more familiar procedures.

(B) For the past half-century or more, there has existed a growing interest in the mathematical analysis of electrical networks. As the field has grown wider and more complex, and circuits have become more complicated, the amount of calculation and effort needed for solution of networks has increased many times. Today, there are in existence several electronic and electro-mechanical devices in laboratories and institutions which have been expressly designed for the purpose of analyzing the more involved circuits. Such machines can do the work of many individual mathematicians, who could conceivably spend hundreds

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Note: Because of time lost during the period when major effort would normally be applied to thesis work, resulting from a serious family illness, this study of the newly-introduced chain-relaxation method has been accepted as a master's thesis, in lieu of a more comprehensive project which had originally been chosen.

of man-hours on a single problem, in a matter of minutes. It is indeed fortunate that such advances have been made, and such analyzing mechanisms will undoubtedly continue to be improved, but they are tremendously expensive items of equipment and require highly-specialized personnel for their operation. Unless a problem is of rather great importance and wide application, it is not profitable to devise a computing mechanism for its solution and recourse must be made to classical methods of computation.

These classical methods of network analysis of lumped-parameter circuits are usually designated as the "branch currents", "mesh-" or "loop-currents", and "nodal" methods. They have in common the technique of forming sets of simultaneous linear equations on the basis of the application of certain elementary laws of circuit behavior. These laws are: Kirchhoff's first rule, which states that the sum of the currents approaching and leaving a node must equal zero, Kirchhoff's second rule, which states that the sum of the voltage drops around a closed path must be equal to zero, and Ohm's law, modified, which states that the current in any branch of a network is equal to the product of the admittance of the branch and the net voltage across the branch. As normally written, these laws are:

(a) $\Sigma I = 0$, at a node,

(b) $\Sigma V = 0$, for a closed path,

(c) $\Sigma YV = I$

or

$\Sigma ZI = E$, for a branch,

where the definitions to be found in Guillemin, Bode, etc., (see Bibliography), for loop, node, and branch are intended. In these equations I

is the current in a branch, V is the potential of a node at which it terminates, and E is the electromotive force of any generator in the branch plus the electromotive forces produced in the branch by the action of vacuum tubes or mutual inductances.

The primary factor in the choice of method of attack of a problem of other than extreme simplicity is the number of unknowns that will have to be determined. For each of the three methods mentioned, criteria have been developed (Bibliography 2, 3, 4, 7) which permit determination of the number of equations which can be expected to develop following the application of the method to a given network. If the number of branches is <u>b</u>, and the number of nodes is <u>n</u>, then in the branch-currents method, there are <u>n-1</u> equations resulting from the application of Kirchhoff's first law to <u>n-1</u> nodes plus <u>b</u> equations resulting from the application of Ohm's law, modified, for each branch. Therefore <u>b + n - 1</u> equations result.

For the mesh-currents method, the resulting number of equations is <u>b - (n - 1)</u>, and for the nodal method the number of equations is (n - 1). (A very good section on this topic is found in Bode's (Bibliography 3) first chapter.)

It is evident that the branch-current method will give the greatest number of equations and it is for this reason that it has been almost entirely discarded except for very simple configurations. The mesh and nodal methods are those in general use today, and are the ones to be considered here.

The mesh-currents method was formulated by Maxwell late in the last century and is perhaps more widely known than the nodal method,

which has achieved popularity within comparatively recent years. Each has advantages and disadvantages. As an example of the variation which can occur in the number of equations, Figure (1) of Plate I shows a circuit which by loop methods might require eight equations (assuming the branches shown could not be simplified) as against three by the nodal method. Figure (2) of the same plate shows a circuit requiring two equations by the loop method as against seven by the nodal method. In general, it may be stated that where the number of nodes is to remain fixed and branches added or taken away, as in an experimental set-up, the nodal analysis is preferable, as the number of equations remains unchanged, requiring only a change in rows or columns of the system determinant if branches are added or taken away. Similarly, in a circuit in which portions of branches are to be varied, keeping the number of meshes constant, the mesh-current method is preferable. The nodal method is currently finding its widest use in analyzing vacuum tube circuits and in many cases results in considerable reduction in the amount of labor required for solution.

There does exist, however, one serious drawback to the nodal analysis. The present procedures cannot be applied if the network contains an arbitrarily located generator of negligible internal impedance or having regulated terminal voltage, as it becomes impossible to replace the voltage source by an equivalent current source as is required by the nodal method. The manner of replacement is clearly presented in many texts (see, for example, Bibliography 3, 7). A second disadvantage lies in the fact that present methods do not permit writing nodal equations directly from a circuit containing mutual inductance. Preliminary calculations are required.

A concise presentation of the manner of calculation is given in Gardner and Barnes, "Transients in Linear Systems", (Bibliography 4). It is worth mentioning at this point, that this second drawback is possibly on the way toward removal. Reed (Bibliography 6) has worked out and presented a paper in which a straightforward system of writing nodal equations directly from the network, and also of taking into account the presence of voltage generators of negligible internal impedance, is outlined. If we assume that Reed's method is valid, then the mesh and nodal methods may be considered of equal merit, and the choice of methods is dictated by the network characteristics. As before, the method should be chosen which involves the lesser amount of calculation, or which more quickly obtains the desired result.

From the foregoing, it can be seen that having selected one of the two available methods of analysis, and assuming that all possible use has been made of network simplification theorems prior to commencing the analysis, a certain fixed amount of computation will remain. If, therefore, another method of solution were developed which consistently gave fewer unknowns in the majority of problems of the type encountered today, its adoption should permit a great reduction in the amount of effort required for solution of network problems. Such a method has been evolved and presented by Dr. L. Tasny-Tschiassny, in the Journal of the Institute of Electrical Engineers (Bibliography 1).

II. Presentation of the Method.

It is not the intention to here reproduce the article mentioned above, as it is published and available, but an abstract of the salient features will be included in order that the illustrative examples given to effect a partial comparison may have meaning. Let it suffice to say

that the author of the referenced article was inspired by Southwell (Bibliography 5), who introduced the method of "residuals" as the basis for his relaxation methods in stress calculations. The chain-relaxation method is an adaptation of these methods to use in electrical circuit work and provides a straightforward means of solution for most networks of the form commonly occurring in the field of radio and communication as it is today.

The chain-relaxation method may be considered as a "forcing" method, in which as few as possible arbitrary branch currents are assumed and from them are derived node potentials, other branch currents, driving currents of current generators, and e.m.f.'s of voltage generators by the use of Kirchhoff's first rule and Ohm's law, modified. Because of the arbitrary assumption of initial branch currents, it becomes necessary at certain nodes to add "residual" currents in order to make the assumed current and potential distributions physically possible.

A very few basic rules render the actual application of the method simple. If the number of generators is $\underline{g}$ and the number of arbitrary assumptions $\underline{s}$, then the number of residual currents (which are the unknowns) will be given by:

(d) $\underline{r} = \underline{s} - \underline{g}$.

A second requirement of the method is that all branches containing voltage sources be free of impedance. This can be accomplished in one of two ways - by arranging for two branches in series, one containing the generator impedance and the other a zero impedance generator, or by replacing the voltage generator with its equivalent current generator having an impedance branch in parallel. This is illustrated in Figure 3, Plate I.

Since it would be a remote chance that the assumed currents were the actual values, the procedure is repeated s - 1 times with different arbitrary values and the results tabulated. These results will also be incorrect but by the principle of superposition the e.m.f.'s of generators or their driving currents may be made equal to their actual values and the residual currents may be made equal to zero. The values of the s multiplying factors will be unknown, but can be found by solving a set of linear equations which are formed properly to fulfill the requirements.

III. The procedure is best illustrated by an example which will be taken step by step, but, in general, one node is selected for reference and an arbitrary current assigned to a branch terminating thereon. By successively applying Kirchhoff's first rule and Ohm's law, modified, adjacent node potentials and branch currents are derived until it becomes necessary to make an additional assumption before proceeding. Residual currents are assigned at any node whenever the currents meeting there do not comply with Kirchoff's first rule.

Results are most conveniently set down in tabular form. There are four principal points in the procedure:

(a) Schedule of steps. This is formed from the diagram of the circuit and indicates the direction in which the computation is to proceed. A symbol is used to indicate steps of special nature, such as an arbitrary assumption.

(b) The operations table. The schedule of steps is set down as Column 1 of a table. In column 2 is indicated by mathematical formulae the nature of the computation for that step, and in s other columns are placed the results of the computation for the s sets of arbitrary assumptions.

(c) The establishment of the equations. Here the results of the operation table are combined in such a way as to find the values of the s assumed currents that will render residual currents zero and e.m.f.'s or driving currents of generators equal to the known values. This step will be clearer when following an actual example.

(d) The computation of values. The s equations are solved for the necessary values of the originally assumed currents and from these values and the operations table, the values of the other branch currents and the node potentials are computed. The principle of superposition is employed. The expression for any quantity contained in a column of the operation's table is multiplied by the correct value of the multiplying factor for that column and the s products are added.

Considerable simplification is achieved if the initially assumed currents are all considered unity. That is, for the first assumption, consider the current equal unity in that column, and other currents zero, then in second column, second current is one and others are zero, etc. This, however, is not a requisite, as any numerical value could be assigned, and the correct multiplying factor would be determined when the equations were solved.

As an illustrative example, the simple network of Plate II is selected and generalized impedances used in the interest of brevity. In practical applications, numerical values would be inserted at any point at which a saving in computation could be effected. Since the network has four independent nodes, requiring four unknowns, but may be solved with only three mesh currents, the mesh analysis is chosen. The solution follows on succeeding plate for one of the mesh currents, which is in fact also a branch current.

On plate (IV), the circuit is reproduced to permit relettering to show the sequence of steps by the chain-relaxation method. Table I of this plate is the operations table. Arbitrary assumptions are indicated by circles around the numeral, and steps leading to the establishment of the equations by squares. The computation will be carried through for one of the assumed currents which corresponds to that determined by mesh analysis, and it will be shown that the two are equal. Node 0 is taken as reference. The solution follows the plate showing circuit and the two currents are shown to be equal on following sheets.

Step 1. Assign an arbitrary value of one to the current through element Z_6. Insert 1 in column 3, and zero in column 4 of table.

Step 2. The potential at 2 is the voltage rise above the reference node. Insert $1 \times Z_6$ in column 3 and $0 \times Z_6$ in column 4.

Step 3. No further movement can be made without a new assumption. Assume I_3, the current to node 0 through Z_3.

Step 4. The potential of node 4 is the voltage rise above reference or I_3Z_3. Insert $0 \times Z_3$ in column 3 and $1 \times X_3$ in column 4.

The procedure is now apparent and will not be detailed. A few notes are in order, however. At step 6, we do not know that this relationship is true and might be inclined to add a residual current. Since only one residual current will be required, $r = s - g - 1$, it is more advisable to let these currents stand and wait to apply the residual at node 7, as in this way the potential of node 7 can be obtained from previously computed values simply. Had we added a residual at node 4, this simple relationship would not hold since

the expression for I_6 would not contain only the assumed and derived values preceding but the residual as well. In step 11, we satisfy Kirchhoff's first rule by the addition of the residual I_{11}, and in step 12 we set up the final condition required for a solution. The equations are then formed as shown on Plate (V). In order to obtain a solution the residual must equal zero and V_{10} must equal known generator voltage. The value of I_{11} in column 3 is multiplied by I_1 and added to the value for I_{11} in column 4 multiplied by I_3 and the sum equated to zero. Likewise the values of V_{10} in column 3 and 4 are multiplied by I_1 and I_3 respectively and their sum equated to the known E. The solution for I_1 follows and is later shown to be equal to the corresponding current obtained from the mesh analysis. The solution would be completed by solving for I_3 and computing the values of any branch currents or node potentials from the values of columns 3 and 4 as previously discussed by summing the respective products.

IV. Conclusions.

It may appear that the saving is small in this case. Another example is shown on plates (VI) and (VII), and the initial equations are set up. This is a generalized version of a twin-T network used by the author of the original article for illustrative purposes. Here it is apparent that either the mesh or nodal methods would require four unknowns, while the chain-relaxation method requires only two, and therefore is much to be preferred. In general, it will be found that the number of unknowns is at least one less than in either the mesh or nodal method when the circuit is of the type illustrated. The addition of vacuum tubes may but does not necessarily increase the number of unknowns in this and the nodal method, while not

the transfer function would [illegible]

values preceding in the residual [illegible]

Lischoff's first rule [illegible] I_{k} [illegible]

step if we set up the final conditions required [illegible] selection. The

equations are then [illegible]

A relation [illegible] that these zero [illegible] greater [illegible]

[illegible] I_{k} [illegible]

[illegible] I_{k} [illegible] one and one are

[illegible] as [illegible]

[illegible]

[illegible] is [illegible]

to the corresponding [illegible]

solution [illegible] and value

of any [illegible]

[illegible]

IV. Conclusions.

[illegible]

[illegible] III, [illegible] IV [illegible]

[illegible]

of the [illegible]

It is [illegible]

[illegible]

[illegible]

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[illegible]

affecting the mesh method, but this possible disadvantage is offset by the fact that most vacuum tube circuits require more mesh than nodal equations.

The addition of generators may work to the disadvantage of the chain-relaxation method, since the number of unknowns can never be less than the number of generators. The other methods in certain networks require less unknowns as current sources or generators are added.

For the types of circuits most frequently encountered, it appears that the chain-relaxation method can be used to advantage. The convenient tabular form lends itself readily to computation and where it is desired to vary elements in the network, the effects of these variations can be readily determined. The form of the operation table permits the computation of only such values as are needed if the complete solution is not required. In any event, it should be possible to determine by inspection of a circuit diagram prior to any computation whether or not the chain-relaxation procedure will reduce the required number of unknowns below that of the better of the usual methods.

It is this writer's opinion that the chain-relaxation method is worthy of extensive trial and further investigation. As yet, no formal set of rules has been made available covering such variables as the best sequence of steps to take or the most advantageous reference node to select. With practice, these points should present no great difficulty, as the optimum procedure will usually be apparent, but it would be of assistance to those not familiar with the method. It is to be hoped that further work will be forthcoming in this field which, at this time, holds at the very least the promise of becoming an almost universally satisfactory method of network analysis.

affecting the mesh method, but this possible disadvantage is offset by the fact that most vacuum tube circuits require more [illegible] than nodal equations.

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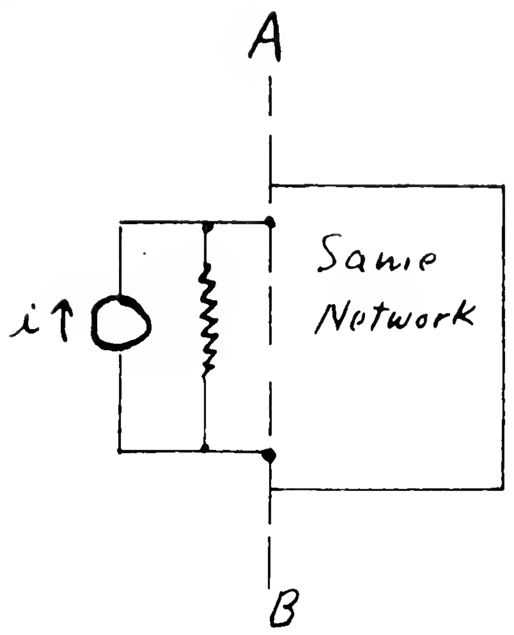
It is true [illegible] working of [illegible] and [illegible] been [illegible] to select. [illegible]

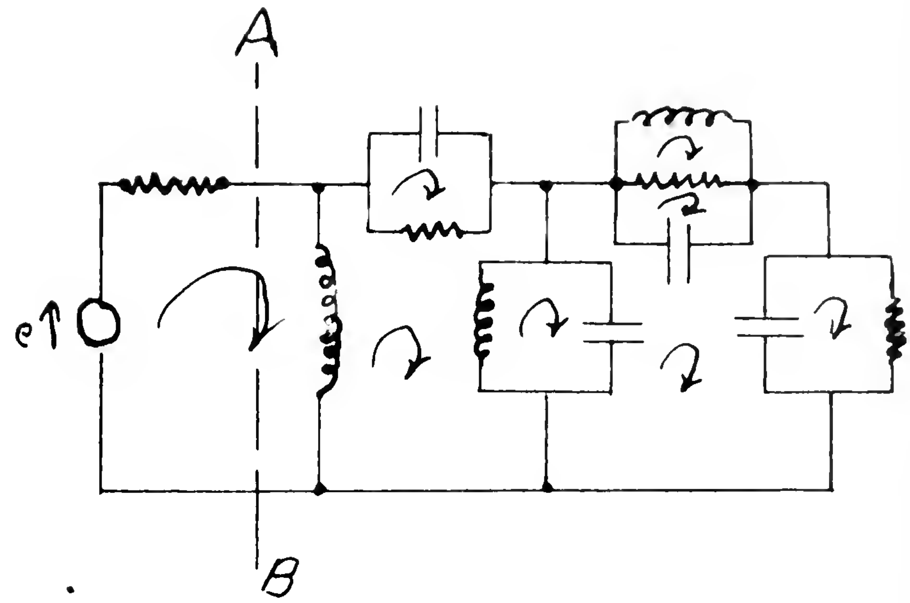


Branches - 12

Nodes - 4

3 Equations by Nodal Method

Figure 1



Branches - 12

Nodes - 5

8 equations by Loop Method

Figure 2

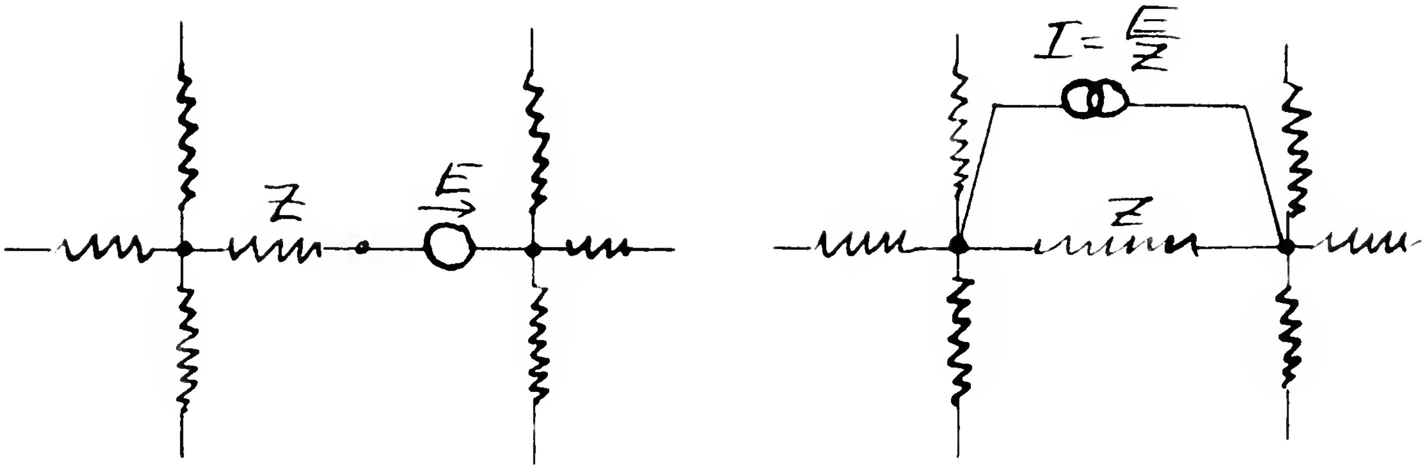
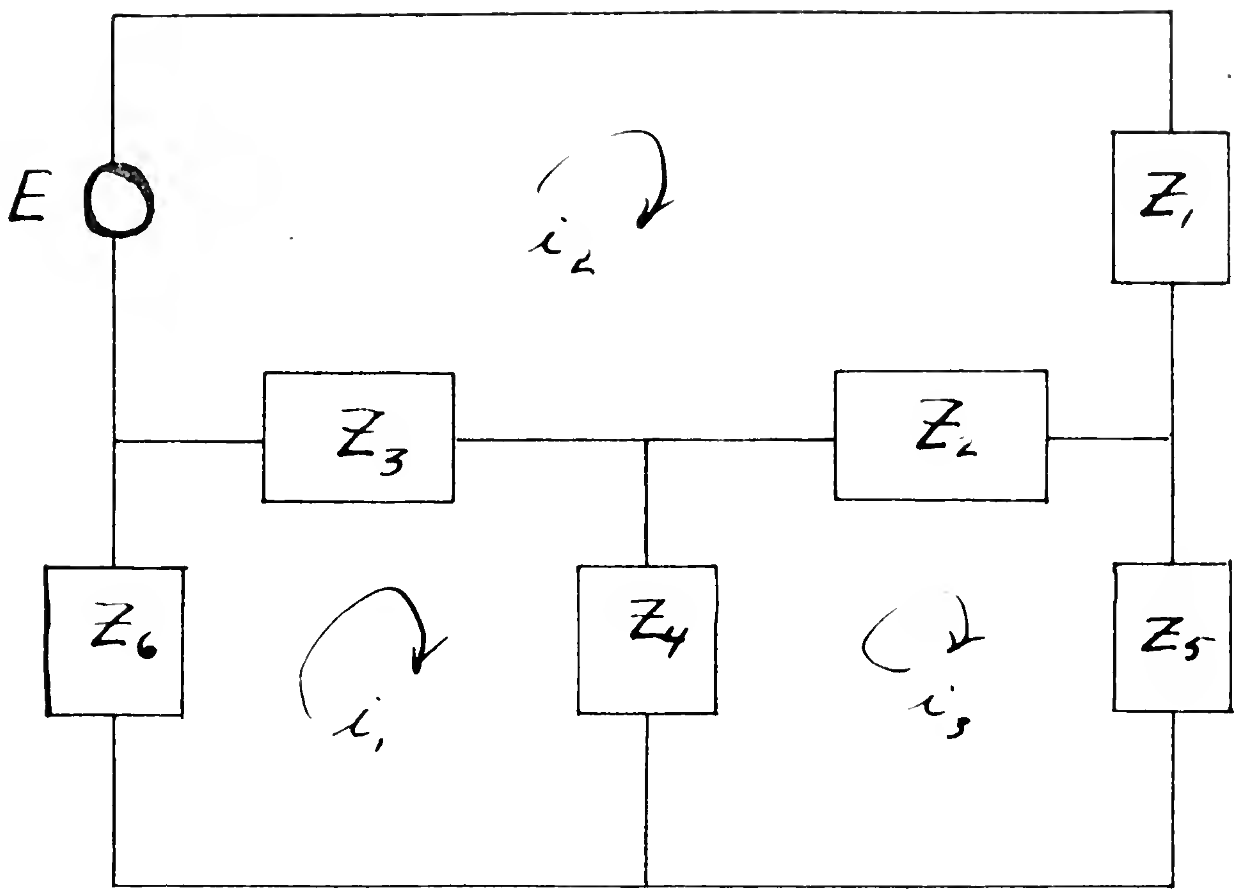


Figure 3

PLATE I



Solution:

$$i_1(Z_3+Z_4+Z_6) - i_2 Z_3 - i_3 Z_4 = 0$$

$$-i_1 Z_3 + i_2(Z_1+Z_2+Z_3) - i_3 Z_2 = E$$

$$-i_1 Z_4 - i_2 Z_2 + i_3(Z_2+Z_4+Z_5) = 0$$

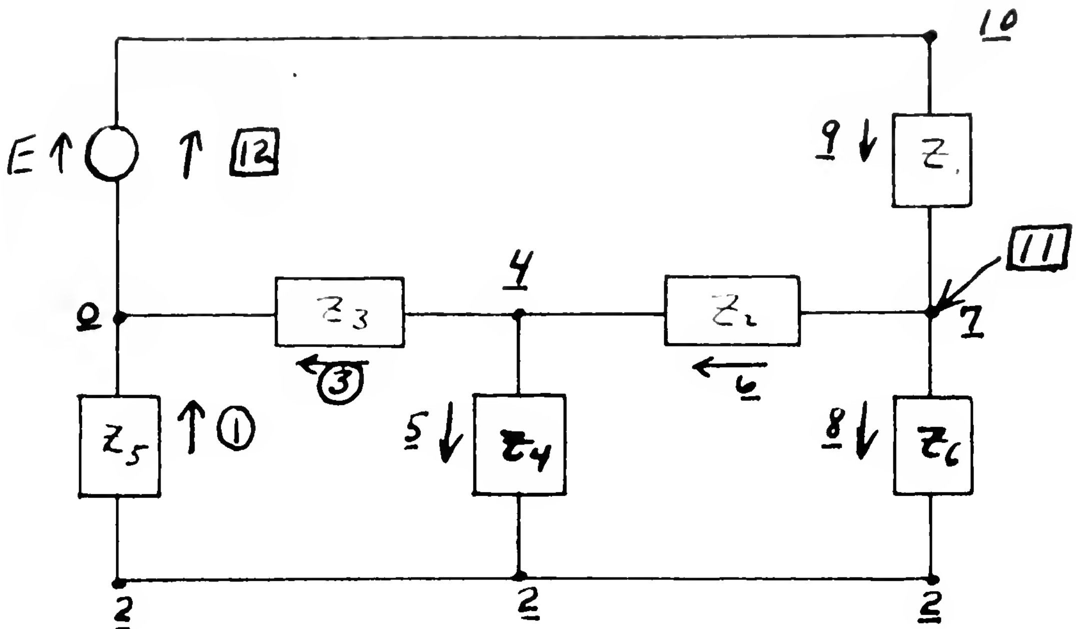
$$i_1 = \frac{\begin{vmatrix} 0 & -Z_3 & -Z_4 \\ E & (Z_1+Z_2+Z_3) & -Z_2 \\ 0 & -Z_2 & (Z_2+Z_4+Z_5) \end{vmatrix}}{\begin{vmatrix} (Z_3+Z_4+Z_6) & -Z_3 & -Z_4 \\ -Z_3 & (Z_1+Z_2+Z_3) & -Z_2 \\ -Z_4 & -Z_2 & (Z_2+Z_4+Z_5) \end{vmatrix}}$$

PLATE II

$$i_1 = \frac{E(z_2 z_4 + z_3 z_2 + z_3 z_4 + z_3 z_5)}{\begin{bmatrix}(z_3+z_4+z_6)(z_1+z_2+z_3)(z_2+z_4+z_5) \\ -(z_2 z_3 z_4) - (z_2 z_3 z_4) - (z_4)^2(z_1+z_2+z_3) \\ -(z_3)^2(z_2+z_4+z_5) - (z_1)^2(z_3+z_4+z_6)\end{bmatrix}}$$

$$i_1 = \frac{E(z_2 z_4 + z_3 z_2 + z_3 z_4 + z_3 z_5)}{\begin{gathered}(z_1 z_2 z_3 + z_1 z_2 z_4 + z_1 z_2 z_6 + z_2 z_3 z_6 \\ + z_1 z_3 z_4 + z_1 z_4 z_6 + z_2 z_4 z_6 + z_3 z_4 z_6 \\ + z_1 z_3 z_5 + z_1 z_4 z_5 + z_2 z_4 z_5 + z_1 z_5 z_6 \\ + z_2 z_5 z_6 + z_3 z_5 z_6 + z_2 z_3 z_5 \\ + z_3 z_4 z_5)\end{gathered}}$$

PLATE III



1	2	3	4
Step No.	Nature of Step	Result	
		for $I_1 = 1,\ I_3 = 0$	for $I_1 = 0,\ I_3 = 1$
1	I_1: arbitrary	1	0
2	$V_2 = I_1 Z_6$	Z_6	0
3	I_3: arbitrary	0	1
4	$V_4 = I_3 Z_3$	0	Z_3
5	$I_5 = \frac{V_4 - V_2}{Z_4}$	$-\frac{Z_6}{Z_4}$	$\frac{Z_3}{Z_4}$
6	$I_6 = I_3 + I_5$	$-\frac{Z_6}{Z_4}$	$1 + \frac{Z_3}{Z_4}$
7	$V_7 = V_4 + I_6 Z_2$	$-\frac{Z_2 Z_6}{Z_4}$	$Z_3 + Z_2 + \frac{Z_2 Z_3}{Z_4}$
8	$I_8 = \frac{V_7 - V_2}{Z_5}$	$-\left(\frac{Z_2 Z_6}{Z_4 Z_5} + \frac{Z_6}{Z_5}\right)$	$\frac{Z_3}{Z_5} + \frac{Z_2}{Z_5} + \frac{Z_2 Z_3}{Z_4 Z_5}$
9	$I_9 = I_1 + I_3$	1	1
10	$V_{10} = V_7 + I_9 Z_1$	$Z_1 - \frac{Z_2 Z_6}{Z_4}$	$Z_1 + Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_4}$
$\boxed{11}$	$I_{11} = I_6 + I_8 - I_9$	$-\left(1 + \frac{Z_6}{Z_4} + \frac{Z_2 Z_6}{Z_4 Z_5} + \frac{Z_6}{Z_5}\right)$	$\frac{Z_3}{Z_4} + \frac{Z_3}{Z_5} + \frac{Z_2}{Z_5} + \frac{Z_2 Z_3}{Z_4 Z_5}$
$\boxed{12}$	$E_{12} = V_{10}$	$Z_1 - \frac{Z_2 Z_6}{Z_4}$	$Z_1 + Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_4}$

PLATE IV

The equations are:

$$-(Z_4Z_5 + Z_6Z_5 + Z_2Z_6)I_1 + (Z_3Z_5 + Z_3Z_4 + Z_2Z_4 + Z_2Z_3)I_3 = 0$$

$$\left(\frac{Z_1Z_4 - Z_2Z_6}{Z_4}\right) I_1 + \left(\frac{Z_1Z_4 + Z_2Z_4 + Z_3Z_4 + Z_2Z_3}{Z_4}\right) I_3 = E$$

$$I_1 = \frac{\begin{vmatrix} 0 & (Z_3Z_5 + Z_3Z_4 + Z_2Z_4 + Z_2Z_3) \\ E & \left(\dfrac{Z_1Z_4 + Z_2Z_4 + Z_3Z_4 + Z_2Z_3}{Z_4}\right) \end{vmatrix}}{\begin{vmatrix} -(Z_4Z_5 + Z_6Z_5 + Z_2Z_4 + Z_4Z_6) & (Z_3Z_5 + Z_3Z_4 + Z_2Z_4 + Z_2Z_3) \\ \left(\dfrac{Z_1Z_4 - Z_2Z_6}{Z_4}\right) & \left(\dfrac{Z_1Z_4 + Z_2Z_4 + Z_3Z_4 + Z_2Z_3}{Z_4}\right) \end{vmatrix}}$$

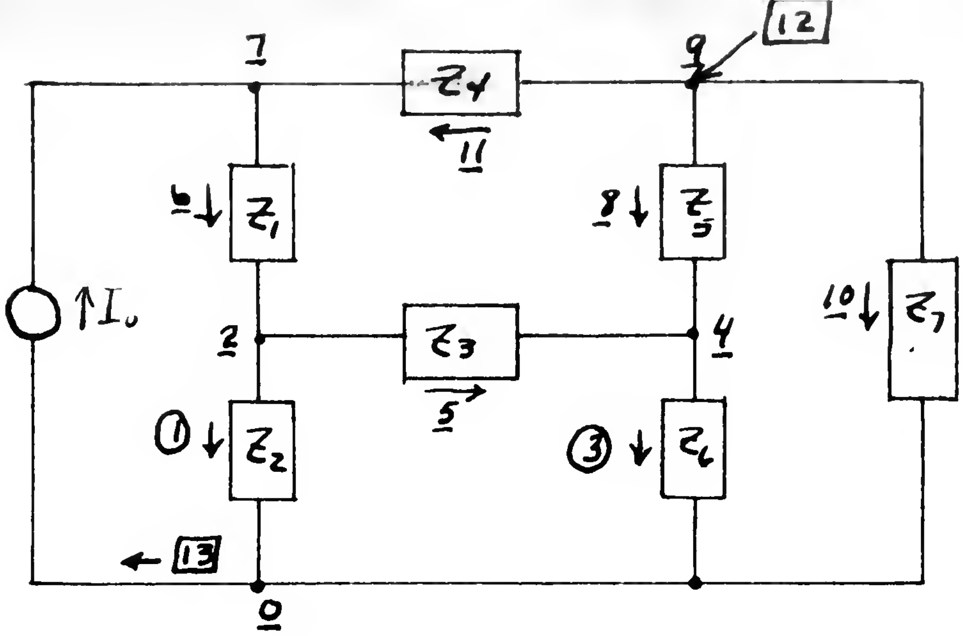
$$I_1 = \frac{-E(Z_3Z_5 + Z_3Z_4 + Z_2Z_4 + Z_2Z_3)Z_4}{-\left[\begin{array}{l}(Z_4Z_5 + Z_6Z_5 + Z_2Z_4 + Z_4Z_6)(Z_1Z_4 + Z_2Z_4 + Z_3Z_4 + Z_2Z_3) \\ \quad + (Z_1Z_4 - Z_2Z_6)(Z_3Z_5 + Z_3Z_4 + Z_2Z_4 + Z_2Z_3)\end{array}\right]}$$

$$I_1 = \frac{E(Z_3Z_5 + Z_3Z_4 + Z_2Z_4 + Z_2Z_3)\cancel{Z_4}}{\begin{array}{l}\cancel{Z_4}(Z_1Z_4Z_5 + Z_4Z_2Z_5 + Z_4Z_3Z_5 + Z_2Z_3Z_5 \\ \quad + Z_1Z_5Z_6 + Z_2Z_5Z_6 + Z_3Z_5Z_6 + Z_1Z_2Z_6 \\ \quad + Z_1Z_4Z_6 + Z_4Z_2Z_6 + Z_3Z_4Z_6 \\ \quad + Z_2Z_3Z_6 + Z_1Z_3Z_5 + Z_1Z_3Z_4 + Z_1Z_2Z_4 \\ \quad + Z_1Z_2Z_3)\end{array}}$$

But this a term-by-term identity with the expression for i_1 on plate III

$$\therefore I_1 = i_1$$

PLATE V



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3	I_3: arbitrary	0	1
4	$V_4 = I_3 Z_6$	0	Z_6
5	$I_5 = \frac{V_2 - V_4}{Z_3}$	$\frac{Z_2}{Z_3}$	$-\frac{Z_6}{Z_3}$
6	$I_6 = I_1 + I_5$	$1 + \frac{Z_2}{Z_3}$	$-\frac{Z_6}{Z_3}$
7	$V_7 = V_2 + I_6 Z_1$	$Z_2 + Z_1\left(1 + \frac{Z_2}{Z_3}\right)$	$-\frac{Z_1 Z_6}{Z_3}$
8	$I_8 = I_3 - I_5$	$-\frac{Z_2}{Z_3}$	$1 + \frac{Z_6}{Z_3}$
9	$V_9 = V_4 + I_8 Z_5$	$-\frac{Z_2 Z_5}{Z_3}$	$Z_6 + Z_5\left(1 + \frac{Z_6}{Z_3}\right)$
10	$I_{10} = \frac{V_9}{Z_7}$	$-\frac{Z_2 Z_5}{Z_3 Z_7}$	$\frac{Z_6}{Z_7} + \frac{Z_5}{Z_7}\left(1 + \frac{Z_6}{Z_3}\right)$
11	$I_{11} = \frac{V_9 - V_7}{Z_4}$	$\frac{1}{Z_4}\left[-\frac{Z_2 Z_5}{Z_3} - Z_2 + Z_1\left(1 + \frac{Z_2}{Z_3}\right)\right]$	$\frac{Z_6}{Z_4} + \frac{Z_5}{Z_4} + \frac{Z_1 Z_6}{Z_3 Z_4}$
$\boxed{12}$	$I_{12} = I_{11} + I_{10} + I_8$	(a)	(b)
$\boxed{13}$	$I_{13} = I_1 + I_3 + I_{10}$	$1 - \frac{Z_2 Z_5}{Z_3 Z_7}$	$1 + \frac{Z_6}{Z_7} + \frac{Z_5}{Z_7}\left(1 + \frac{Z_6}{Z_3}\right)$

$$(a) = \frac{1}{Z_4}\left[-\frac{Z_2 Z_5}{Z_3} - Z_2 + Z_1\left(1 + \frac{Z_2}{Z_3}\right)\right] - \frac{Z_2 Z_5}{Z_3 Z_7} - \frac{Z_2}{Z_3}.$$

$$(b) = \frac{Z_6}{Z_4} + \frac{Z_5}{Z_4} + \frac{Z_1 Z_6}{Z_3 Z_4} + \frac{Z_6}{Z_7} + \frac{Z_5}{Z_7}\left(1 + \frac{Z_6}{Z_3}\right) + 1 + \frac{Z_6}{Z_3}.$$

PLATE VI

The equations to be solved are therefore:

$$\left[\left(\frac{-Z_2Z_5 - Z_2Z_3Z_4 + Z_1Z_3Z_4 + Z_1Z_2Z_4}{Z_3Z_4}\right) - \frac{Z_2Z_5}{Z_3Z_7} - \frac{Z_2}{Z_3}\right] I_1$$

$$+ \left[\frac{Z_6}{Z_4} + \frac{Z_5}{Z_4} + \frac{Z_1Z_6}{Z_3Z_4} + \frac{Z_6}{Z_7} + \frac{Z_5}{Z_7} + \frac{Z_5Z_6}{Z_3Z_7} + 1 + \frac{Z_6}{Z_3}\right] I_3 = 0$$

and

$$\left(1 - \frac{Z_2Z_5}{Z_3Z_7}\right) I_1 + \left[1 + \frac{Z_6}{Z_7} + \frac{Z_5}{Z_7} + \frac{Z_5Z_6}{Z_3Z_7}\right] I_3 = I_0$$

The first equation is homogeneous and can be solved for I_1 in terms of I_3 and then solve for I_3 in the second equation. With these values, the value of any of the quantities in the table can now be computed.

PLATE VII

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